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Solutions Chapitre IV et VExercice 1

$$1) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} ; \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{18} \begin{pmatrix} 8a + 5b - c \\ -2a + b + 7c \\ -4a - 7b + 5c \end{pmatrix}$$

Exercice 2

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

Exercice 3

$$A = \begin{pmatrix} 2 & 4 & 4 \\ 1 & 3 & 1 \\ 4 & 5 & 6 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 3 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 7 \end{pmatrix}$$

Exercice 4

$$① \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$② \quad \det(A) = \det(U) = 8$$

$$③ \quad b_1 = \begin{pmatrix} 1 \\ -2 \\ 3/4 \\ -1/2 \end{pmatrix} \quad b_2 = \begin{pmatrix} 3/2 \\ -5 \\ 5/2 \\ -2 \end{pmatrix}$$

$$\text{Exercice 5} \quad B = LL^T \quad L = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 2 & 1 \end{pmatrix} \quad b_3 = \begin{pmatrix} -6 \\ 5 \\ -3 \\ 1 \end{pmatrix}$$

② Exercices Conditionnement et Normes

Exercice 1

$$\|x_1\|_1 = 6$$

$$\|x_2\|_1 = 10$$

$$\|x_3\|_1 = 10$$

$$\|x_4\|_1 = 17$$

$$\|x_1\|_\infty = 2$$

$$\|x_2\|_\infty = 4$$

$$\|x_3\|_\infty = 5$$

$$\|x_4\|_\infty = 10$$

$$\|x_1\|_2 = \sqrt{10}$$

$$\|x_2\|_2 = \sqrt{30}$$

$$\|x_3\|_2 = \sqrt{38}$$

$$\|x_4\|_2 = \sqrt{127}$$

Exercice 2

1) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2) $N_1 \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2+\varepsilon \\ 2 \end{pmatrix} \right) = \varepsilon$

$N_1 \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} \right) = 4$

$$\frac{N_1 \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2+\varepsilon \\ 2 \end{pmatrix} \right)}{N_1 \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} \right)} = \frac{\varepsilon}{4}$$

3) $\frac{N_1(x-y)}{N_1(x)} = \frac{2}{2} = 1$

4) on a ~~1~~ $1 \leq \text{cond}(A) \times \frac{\varepsilon}{4} \Rightarrow \text{cond}(A) \geq \frac{4}{\varepsilon}$

5) of cows -