Correction Partiel INSA novembre 2006

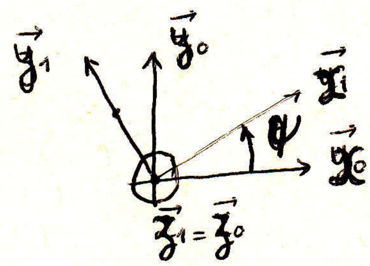
Support : centrifugeuse humaine

Partie 1 : Torseur cinétique

Question 1 le plan $(O\,\vec{y_1}\,\vec{z_1})$ est plan de symétrie donc $(G_1, \vec{y_1}, \vec{z_1})$ aussi car $\vec{OG_1} = a\vec{y_1}$ est dans ce plan.

on déduit $E_1 = F_1 = 0$ d'où la matrice $[I_{G_1}(1)] = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1 \end{pmatrix}$

Question 2 $\{\mathcal{C}_i(1/0)\} = \begin{Bmatrix} \vec{R_c}(1/0) \\ \vec{\sigma_O}(1/0) \end{Bmatrix}_O$



$\vec{R_c}(1/0) = m_1 \vec{V}_{G_1,1/0}$

$\vec{V}_{G_1,1/0} = \left.\dfrac{d(a\vec{y_1})}{dt}\right|_0 = -a\dot{\psi}\vec{x_1}$

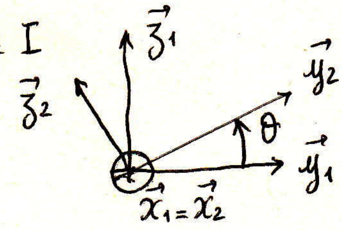
$\vec{R_c}(1/0) = -m_1 a\dot{\psi}\vec{x_1}$

$\vec{\sigma_O}(1/0) = [I_O(1)]\{\Omega 1/0\}$ avec Huyghens $[I_O(1)] = \begin{pmatrix} A_1+m_1a^2 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1+m_1a^2 \end{pmatrix}$

$$= \begin{pmatrix} A_1+m_1a^2 & 0 & 0 \\ 0 & B_1 & -D_1 \\ 0 & -D_1 & C_1+m_1a^2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 0 \\ -D_1\dot{\psi} \\ (C_1+m_1a^2)\dot{\psi} \end{pmatrix}_{b_1}$$

Question 3 Anneau - 2 plans de symétrie passant par $G_2 \equiv I$

d'où $[I_I(2)] = \begin{pmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{pmatrix}$



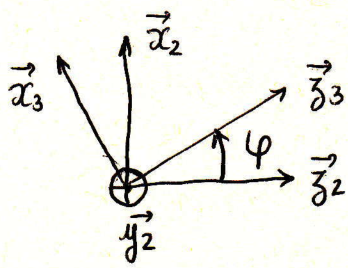
Question 4 $\{\mathcal{C}_i(2/0)\} = \begin{Bmatrix} \vec{R_c}(2/0) \\ \vec{\sigma_I}(2/0) \end{Bmatrix}_I$ $\quad \vec{R_c}(2/0) = m_2\vec{V}_{I,2/0} = +m_2R\dot{\psi}\vec{x_1}$

car $\vec{V}_{I,2/0} = \dfrac{d(-R\vec{y_1})}{dt} = +R\dot{\psi}\vec{x_1}$

$\vec{\sigma}_{I,(2/0)} = [I_{I,(2)}](\vec{\Omega}2/0)$

$\vec{\Omega}_{2/0} = \begin{vmatrix} \dot{\theta} \\ \dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta \end{vmatrix}_{b_2}$ $\quad \vec{\sigma}_{I,2/0} = \begin{pmatrix} A_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & C_2 \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta \end{pmatrix} = \begin{pmatrix} A_2\dot{\theta} \\ B_2\dot{\psi}\sin\theta \\ C_2\dot{\psi}\cos\theta \end{pmatrix}_{b_2}$

Question 5 torseur cinétique $\left\{\mathcal{C}_i(3/0)\right\} = \left\{\begin{matrix}\vec{R_c}(3/0)\\ \vec{\sigma}_{I,3/0}\end{matrix}\right\}_I$



$$\vec{R_c}(3/0) = m_3 \vec{V}_{G3,3/0} = m_3 \vec{V}_{I,3/0} = m_3 R \dot{\psi} \vec{x_1}$$

$$\vec{\Omega}(3/0) = \dot{\psi}\vec{z_1} + \dot{\theta}\vec{x_2} + \dot{\varphi}\vec{y_2} = \begin{pmatrix}\dot{\theta}\\ \dot{\varphi} + \dot{\psi}\sin\theta\\ \dot{\psi}\cos\theta\end{pmatrix}_{b_2}$$

$$\vec{\sigma}_{I,3/0} = \begin{pmatrix}A_3 & 0 & 0\\ 0 & B_3 & 0\\ 0 & 0 & C_3\end{pmatrix}_{b_2}\begin{pmatrix}\dot{\theta}\\ \dot{\varphi} + \dot{\psi}\sin\theta\\ \dot{\psi}\cos\theta\end{pmatrix}_{b_2} = \begin{pmatrix}A_3\dot{\theta}\\ B_3(\dot{\varphi} + \dot{\psi}\sin\theta)\\ C_3\dot{\psi}\cos\theta\end{pmatrix}_{b_2}$$

Question 6 $\left\{\mathcal{C}_i(E/0)\right\} = \left\{\begin{matrix}\vec{R_c}(2/0) + \vec{R_c}(3/0)\\ \vec{\sigma}_I(2/0) + \vec{\sigma}_I(3/0)\end{matrix}\right\}_I$

Il suffit d'additionner les expressions trouvées précédemment

$$\vec{R_c}(E/0) = (m_2 + m_3) R \dot{\psi} \vec{x_1} \qquad \vec{\sigma}_I(E/0) = \begin{pmatrix}(A_2 + A_3)\dot{\theta}\\ (B_2 + B_3)\dot{\psi}\sin\theta + B_3\dot{\varphi}\\ (C_2 + C_3)\dot{\psi}\cos\theta\end{pmatrix}_{b_2}$$

Partie 2 Torseur dynamique

Question 7 $\left\{\mathcal{D}(1/0)\right\} = \left\{\begin{matrix}\vec{R_d}(1/0) = m_1 \vec{\gamma}_{G1,1/0}\\ \vec{\delta}_O(1/0)\end{matrix}\right\}_O$

$$\vec{R_d}(1/0) = m_1 a \ddot{\psi} \vec{x_1} - m_1 a \dot{\psi}^2 \vec{y_1}$$

O est point fixe $\vec{\delta}_O(1/0) = \dfrac{d\vec{\sigma}_O(1/0)}{dt} = \dfrac{d}{dt}\left\{-D_1\dot{\psi}\vec{y_1} + (C_1 + m_1 a^2)\dot{\psi}\vec{z_1}\right\}$

$$= -D_1\ddot{\psi}\vec{y_1} + D_1\dot{\psi}^2\vec{x_1} + (C_1 + m_1 a^2)\ddot{\psi}\vec{z_1}$$

Question 8 $\qquad \{\mathcal{D}(3/0)\} = \begin{Bmatrix} \vec{R_d}(3/0) \\ \vec{\delta_I}(3/0) \end{Bmatrix}_I$

$$\vec{R_d}(3/0) = \left.\frac{d\vec{R_c}(3/0)}{dt}\right)_{/0} = \frac{d}{dt}\left\{m_3 R\dot{\psi}\vec{x_1}\right\} = m_3 R\ddot{\psi}\vec{x_1} + m_3 R\dot{\psi}^2\vec{y_1}$$

$$\vec{\delta_I}(3/0) = \left.\frac{d\vec{\sigma_I}(3/0)}{dt}\right)_{/0}$$ car I est centre de gravité de (3)

$$\left.\frac{d\vec{\sigma}_{I,3/0}}{dt}\right)_{/0} = \left.\frac{d\vec{\sigma}_{I,3/0}}{dt}\right)_{/2} + \left\{\vec{\Omega}_{2/0}\wedge\vec{\sigma}_{I,3/0}\right\} = \begin{pmatrix} A_3\ddot{\theta} \\ B_3(\ddot{\varphi}+\ddot{\psi}\sin\theta+\dot{\psi}\dot{\theta}\cos\theta) \\ C_3(\ddot{\psi}\cos\theta-\dot{\psi}\dot{\theta}\sin\theta) \end{pmatrix}_{b_2} + \begin{pmatrix} \dot{\theta} \\ \dot{\varphi}+\dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta \end{pmatrix} \wedge \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$= \begin{pmatrix} A_3\ddot{\theta} + (\dot{\varphi}+\dot{\psi}\sin\theta)C_3\dot{\psi}\cos\theta - (\dot{\psi}\cos\theta)B_3(\dot{\varphi}+\dot{\psi}\sin\theta) \\ B_3(\ddot{\varphi}+\ddot{\psi}\sin\theta+\dot{\psi}\dot{\theta}\cos\theta) + (\dot{\psi}\cos\theta)A_3\dot{\theta} - \dot{\theta}(C_3\dot{\psi}\cos\theta) \\ C_3(\ddot{\psi}\cos\theta-\dot{\psi}\dot{\theta}\sin\theta) + \dot{\theta}\cdot B_3(\dot{\varphi}+\dot{\psi}\sin\theta) - (\dot{\varphi}+\dot{\psi}\sin\theta)A_3\dot{\theta} \end{pmatrix}_{b_2}$$

Question 9 $\qquad \vec{\delta}_{O}(E/0) \qquad E \equiv (1)+(2)+(3)$

$$\boxed{\vec{\delta}_{O,E/0} = \vec{\delta}_{O,1/0} + \vec{\delta}_{O,2/0} + \vec{\delta}_{O,3/0}}$$

$\boxed{\vec{\delta}_{O,1/0}}$ (Question 7)

$$\boxed{\vec{\delta}_{O,2/0} = \vec{\delta}_{I,2/0} + \{\vec{OI}\wedge\vec{R_d}(2/0)\}}$$

$$\boxed{\vec{\delta}_{I,2/0} = \frac{d\vec{\sigma}_{I,2/0}}{dt}}$$

$\boxed{\vec{\sigma}_{I,2/0}}$ (Question 4)

$$\boxed{\vec{R_d}(2/0) = \frac{d\vec{R_c}(2/0)}{dt}}$$

obtenu en remplaçant m_3 par m_2 dans $\vec{R_d}(3/0)$ (Question 8)

$$\boxed{\vec{\delta}_{O,3/0} = \vec{\delta}_{I,3/0} + \{\vec{OI}\wedge\vec{R_d}(3/0)\}}$$

$\boxed{\vec{\delta}_{I,3/0}}$ (Question 8)

$\boxed{\vec{R_d}(3/0)}$ (Question 8)