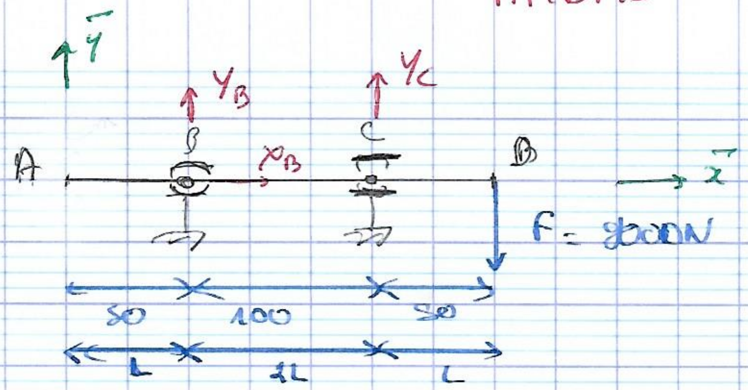
FLEXION : RIGIDITE ET RESISTANCE D'UN ARBRE

1. Modélisation :

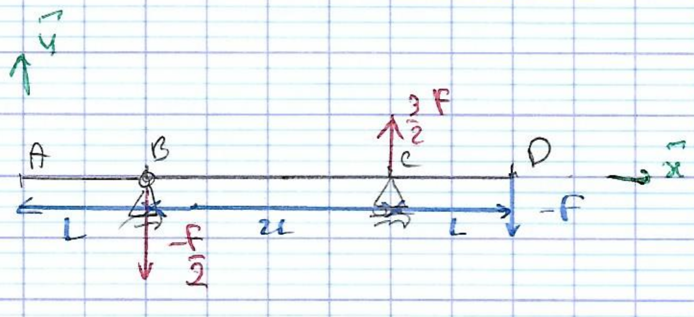


PFS

$$\begin{cases} X_B = 0 & (1) \\ Y_B + Y_C - F = 0 & (2) \\ 2L\,Y_C - 3LF = 0 & (3) \end{cases}$$

(3) $\Rightarrow \quad Y_C = \dfrac{3F}{2}$

(2) $\Rightarrow \quad Y_B = F - \dfrac{3F}{2} = \dfrac{-F}{2}$



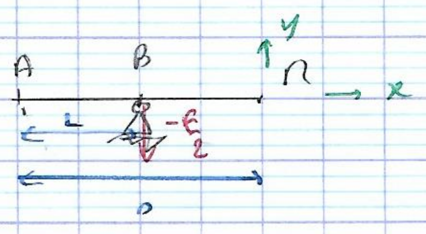
2. Efforts intérieurs

Coupure 1 : $M \in [AB]$, $0 \le x \le L$

$$\{\mathcal{T}_{coh\ int\ o/c}\}_M = -\{\mathcal{T}_{ext \to o}\}_M = -\begin{cases} \vec{R}_M = \vec{0} \\ \vec{M}_M = \vec{0}. \end{cases}$$

Sollicitation : $\emptyset$.

Coupure 2 : $M \in [BC]$, $L \leq x \leq 3L$.

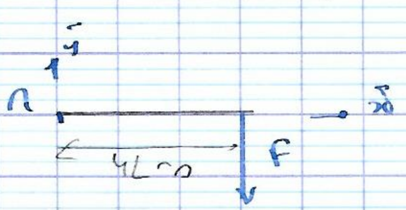


$$\{\mathcal{T}_{act\ int\ M/G}\}_M = -\{\mathcal{T}_{ext \to G}\}_M = \begin{cases} \vec{S}_M = -\frac{F}{2}\vec{j} \\ \vec{M}_M = \frac{F}{2}(x-L)\vec{k} \end{cases}$$

Sollicitations :

$$\begin{cases} T_y = \frac{F}{2} \\ M_{f_z} = -\frac{F}{2}(x-L). \end{cases}$$

Coupure 3 : $M \in [CD]$. $3L \leq x \leq 4L$

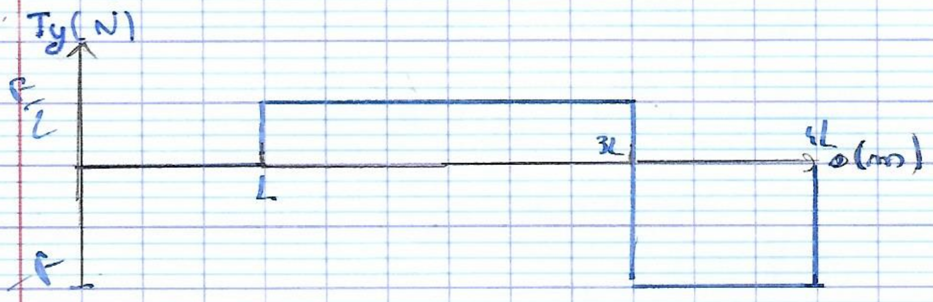


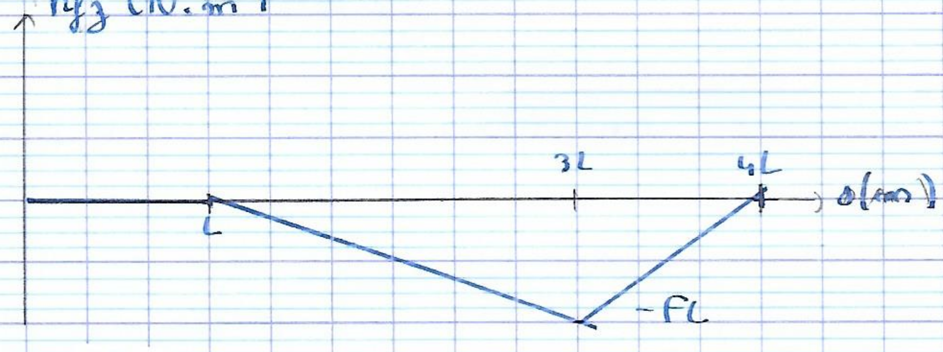
$$\{\mathcal{T}_{act\ int\ M/G}\}_M = \{\mathcal{T}_{ext \to G}\}_M = \begin{cases} \vec{S}_M = -F\vec{j} \\ \vec{M}_M = -F(4L-x)\vec{k} \end{cases}$$

Sollicitation :

$$\begin{cases} T_y = -F \\ M_{f_z} = -F(4L-x). \end{cases}$$

Diagrammes :





3_Contraintes

$$\sigma_{max} = \frac{Mfz_{max}}{I_z} \times \frac{D}{2} = \frac{32\,Mfz_{max}}{\pi D^3}.$$

$$\sigma_{max} < \frac{Re}{Cs}$$

Matériaux : Acier S235

$Re = 235\,MPa$.

$$\frac{32\,Mfz_{max}}{\pi D^3} < \frac{Re}{Cs} \iff D > \sqrt[3]{\frac{32FL.Cs}{\pi Re}}$$

A.N : $D > \sqrt[3]{\dfrac{32 \cdot 9000 \cdot 4 \cdot 1{,}5}{\pi \cdot 235 \cdot 10^6}}$

$\boxed{D > 31\,mm}$

4 _ Critères.

flèche

$$f = \frac{PL^2(3L-L)}{3EI_z}$$

$$f = \frac{4FL^3 \cdot 64}{3E\pi D^4} < \bar{f} = 0{,}1\,mm.$$

$$D > \sqrt[4]{\frac{256 \cdot FL^3}{3E\pi \bar{f}}}$$

A.N: $$D > \sqrt[4]{\frac{256 \cdot 9000 \cdot (80 \cdot 10^{-3})^3}{3 \cdot 216 \cdot 10^9 \cdot \pi \cdot 0{,}1 \cdot 10^{-3}}}$$

$$\boxed{D > 35\,mm}.$$

On prend donc $D = 35\,mm$.

Angle de rotulage

$$\alpha_i = \frac{-F \times L\,(6L + 3L)}{EI_z}$$

$$\alpha_i = 2 \cdot 10^{-3}\,° < 2°$$

$\alpha_i < \alpha_{max}$

Le critère est vérifié